

# COLLEGE ROUND ONE

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You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, you must do so before the next problem is displayed by notifying a grader.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

# COLLEGE ROUND ONE



**All answers must be simplified which means:**

- Rational numbers must be in reduced form/simplified.

BAD:  $\frac{4}{2}$  vs GOOD: 2 or BAD:  $\frac{1}{3} + \frac{1}{6}$  vs GOOD:  $\frac{1}{2}$

- When irrational number(s) (e.g.  $e$ ,  $\pi$ ,  $\ln 3$ ) are involved in fractions, you do NOT need to write as a single quantity and do not need to combine multiple logs.

$\frac{1}{e} + 2$  is just as good as  $\frac{1+2e}{e}$ ,  $2 \ln 3$  is just as good as  $\ln 9$

However, NO negative powers should remain. So  $e^{-1}$  will be marked incorrect.

- All trig functions must be evaluated.

# COLLEGE ROUND ONE

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At most five participants will move to the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary.

**Everyone moving to the finals will receive \$25.**

Questions?

INTEGRAL #1

**READY,  
GET SET,...**

**2:00**

INTEGRAL #1

$$\int_0^1 (1600x^3 + 300x^2 + 1000x + 1017) dx$$

**INTEGRAL #1**

$$\int_0^1 (1600x^3 + 300x^2 + 1000x + 1017) dx$$

$$= \left[ 1600 \cdot \frac{x^4}{4} + 300 \cdot \frac{x^3}{3} + 1000 \cdot \frac{x^2}{2} + 1017x \right]_0^1$$

$$= \left[ 400x^4 + 100x^3 + 500x^2 + 1017x \right]_0^1$$

$$= \mathbf{2017}$$

**INTEGRAL #2**

**READY,  
GET SET,...**

**2:00**

INTEGRAL #2

$$\int_0^{\pi/3} \frac{\sec x \tan x}{\sqrt{2 + \sec x}} dx$$



**INTEGRAL #2**

$$\int_0^{\pi/3} \frac{\sec x \tan x}{\sqrt{2 + \sec x}} dx$$

$$= \int_3^4 \frac{1}{\sqrt{u}} du \quad u = 2 + \sec x, \quad du = \sec x \tan x dx$$

$$= \left[ 2\sqrt{u} \right]_3^4$$

$$= 4 - 2\sqrt{3}$$

INTEGRAL #3

**READY,  
GET SET,...**

**2:00**

INTEGRAL #3

$$\int_1^{25} \frac{\sqrt{5 - \sqrt{x}}}{\sqrt{x}} dx$$

**INTEGRAL #3**

$$\int_1^{25} \frac{\sqrt{5 - \sqrt{x}}}{\sqrt{x}} dx$$

$$= 2 \int_4^0 \sqrt{u} du \quad u = 5 - \sqrt{x}, \quad du = -\frac{1}{2\sqrt{x}} dx$$

$$= \frac{4}{3} \left[ u^{3/2} \right]_0^4$$

$$= \frac{32}{3}$$

INTEGRAL #4

**READY,  
GET SET,...**

**2:00**

INTEGRAL #4

$$\int_0^{\ln \sqrt{3}} \frac{e^x}{e^{2x} + 1} dx$$

**INTEGRAL #4**

$$\int_0^{\ln \sqrt{3}} \frac{e^x}{e^{2x} + 1} dx$$

$$= \int_1^{\sqrt{3}} \frac{1}{u^2 + 1} du$$

$$u = e^x, \quad du = e^x dx$$

$$= \left[ \arctan u \right]_1^{\sqrt{3}}$$

$$= \boxed{\frac{\pi}{12}}$$

INTEGRAL #5

**READY,  
GET SET,...**

**2:00**



INTEGRAL #5

$$\int_0^{13} \frac{1}{\sqrt[3]{2x+1}} dx$$

**INTEGRAL #5**

$$\int_0^{13} \frac{1}{\sqrt[3]{2x+1}} dx$$

$$= \frac{1}{2} \int_1^{27} u^{-1/3} du \quad u = 2x + 1, \quad du = 2 dx$$

$$= \frac{3}{4} \left[ u^{2/3} \right]_1^{27}$$

$$= \boxed{6}$$

INTEGRAL #6

**READY,  
GET SET,...**

**2:00**

INTEGRAL #6

$$\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx$$

**INTEGRAL #6**

$$\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx$$

$$= \int_0^{\pi/2} \cos x (1 - \sin^2 x) \sin^2 x \, dx$$

$$= \int_0^1 (1 - u^2) u^2 \, du \quad u = \sin x, \quad du = \cos x \, dx$$

$$= \left[ \frac{u^3}{3} - \frac{u^5}{5} \right]_0^1 = \boxed{\frac{2}{15}}$$

INTEGRAL #7

**READY,  
GET SET,...**

**2:00**

INTEGRAL #7

$$\int_0^9 \sqrt[9]{x^2} \cdot \sqrt{x^5} dx$$

**INTEGRAL #7**

$$\int_0^9 \sqrt[9]{x^2} \cdot \sqrt{x^5} dx$$

$$= \int_0^9 x^{1/2} dx$$

$$= \left[ \frac{2x^{3/2}}{3} \right]_0^9$$

$$= \mathbf{18}$$



INTEGRAL #8

**READY,  
GET SET,...**

**2:00**

INTEGRAL #8

$$\int_1^e 16x^3 \ln x \, dx$$

**INTEGRAL #8**

$$\int_1^e 16x^3 \ln x \, dx$$

integrate by parts:      $u = \ln x$       $dv = 16x^3 \, dx$   
                                  $du = \frac{1}{x} \, dx$       $v = 4x^4$

$$= \left[ 4x^4 \ln x \right]_1^e - 4 \int_1^e x^3 \, dx$$

$$= \left[ 4x^4 \ln x - x^4 \right]_1^e$$

$$= \boxed{3e^4 + 1}$$

INTEGRAL #9

**READY,  
GET SET,...**

**2:00**

INTEGRAL #9

$$\int_0^{\pi/2} \cos x \cdot \sin(\pi \sin x) dx$$

**INTEGRAL #9**

$$\int_0^{\pi/2} \cos x \cdot \sin(\pi \sin x) dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin u du \quad u = \pi \sin x, \quad du = \pi \cos x dx$$

$$= -\frac{1}{\pi} [\cos u]_0^{\pi}$$

$$= \frac{2}{\pi}$$

INTEGRAL #10

**READY,  
GET SET,...**

**2:00**

INTEGRAL #10

$$\int_0^1 \frac{2x + 3}{(x + 1)(x + 2)} dx$$



**INTEGRAL #10**

$$\int_0^1 \frac{2x + 3}{(x + 1)(x + 2)} dx$$

$$= \int_0^1 \left( \frac{1}{x + 1} + \frac{1}{x + 2} \right) dx$$

partial fractions

$$= \left[ \ln|x + 1| + \ln|x + 2| \right]_0^1$$

$$= \ln 3$$

INTEGRAL #11

**READY,  
GET SET,...**

**2:00**

INTEGRAL #11

$$\int_1^3 \left( \frac{2}{\sqrt{x}} + \sqrt{x} \right)^2 dx$$

**INTEGRAL #11**

$$\int_1^3 \left( \frac{2}{\sqrt{x}} + \sqrt{x} \right)^2 dx$$

$$= \int_1^3 \left( \frac{4}{x} + 4 + x \right) dx$$

$$= \left[ 4 \ln|x| + 4x + \frac{x^2}{2} \right]_1^3$$

$$= 12 + 4 \ln 3 \quad \text{or} \quad 12 + \ln 81$$

INTEGRAL #12

**READY,  
GET SET,...**

**2:00**

INTEGRAL #12

$$\int_0^1 \frac{2x^2 + 2x + 1}{x + 1} dx$$

**INTEGRAL #12**

$$\begin{aligned} & \int_0^1 \frac{2x^2 + 2x + 1}{x + 1} dx \\ &= \int_0^1 \left( \frac{2x^2 + 2x}{x + 1} + \frac{1}{x + 1} \right) dx \\ &= \int_0^1 \left( 2x + \frac{1}{x + 1} \right) dx \\ &= \left[ x^2 + \ln|x + 1| \right]_0^1 \\ &= \mathbf{1 + \ln 2} \end{aligned}$$

INTEGRAL #13

**READY,  
GET SET,...**

**2:00**



INTEGRAL #13

$$\int_{-1}^2 \frac{1}{x^2 + 2x + 10} dx$$

**INTEGRAL #13**

$$\int_{-1}^2 \frac{1}{x^2 + 2x + 10} dx$$

$$= \int_{-1}^2 \frac{1}{(x + 1)^2 + 9} dx$$

complete the square

$$= \left[ \frac{1}{3} \arctan \left( \frac{x + 1}{3} \right) \right]_{-1}^2$$

$$= \frac{\pi}{12}$$

INTEGRAL #14

**READY,  
GET SET,...**

**2:00**

INTEGRAL #14

$$\int_0^{\pi/3} \frac{\sec x + \sin x}{\cos x} dx$$

**INTEGRAL #14**

$$\int_0^{\pi/3} \frac{\sec x + \sin x}{\cos x} dx$$

$$= \int_0^{\pi/3} \left( \frac{\sec x}{\cos x} + \frac{\sin x}{\cos x} \right) dx$$

$$= \int_0^{\pi/3} (\sec^2 x + \tan x) dx = \left[ \tan x - \ln|\cos x| \right]_0^{\pi/3}$$

$$= \sqrt{3} - \ln \frac{1}{2} \quad \text{or} \quad \sqrt{3} + \ln 2$$

INTEGRAL #15

**READY,  
GET SET,...**

**2:00**

INTEGRAL #15

$$\int_0^{\pi} \frac{\sin 2x}{2 + \cos x} dx$$

**INTEGRAL #15**

$$\int_0^{\pi} \frac{\sin 2x}{2 + \cos x} dx$$

$$= 2 \int_0^{\pi} \frac{\sin x \cos x}{2 + \cos x} dx \quad \sin 2x = 2 \sin x \cos x$$

$$= -2 \int_3^1 \frac{u - 2}{u} du \quad u = 2 + \cos x, \quad du = -\sin x dx$$

$$= 2 \left[ u - 2 \ln|u| \right]_1^3$$

$$= 4 - 4 \ln 3 \quad \text{or} \quad 4 - \ln 81$$