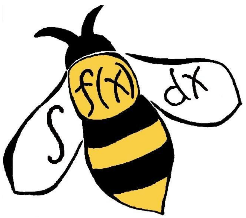
HIGH SCHOOL ROUND ONE



You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, you must do so before the next problem is displayed by notifying a grader.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.

⇛

HIGH SCHOOL ROUND ONE

All answers must be simplified which means

- Rational numbers must be simplified and reduced.

  $\frac{4}{2}$ ✗ vs 2 ✓ $\qquad$ $\frac{1}{3}+\frac{1}{6}$ ✗ vs $\frac{1}{2}$ ✓

- Trig functions must be evaluated when possible.

  $\sin\frac{\pi}{3}$ ✗ vs $\frac{\sqrt{3}}{2}$ ✓ $\qquad$ $\tan 0$ ✗ vs 0 ✓

- You do **not** need to combine multiple logs or fractions with irrational numbers (e.g. π, e, $\ln 3$).

  $\frac{1}{e}+2$ ✓, $\frac{1+2e}{e}$ ✓ $\qquad$ $2\ln 3$ ✓, $\ln 9$ ✓

$\Rrightarrow$

HIGH SCHOOL ROUND ONE

- However, **no** negative powers should remain.

  e^{-1} ✗ vs $\frac{1}{e}$ ✓

At most five participants will move onto the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary.

Everyone moving onto the finals will receive $25.

READY,

GET SET,...

2:00

INTEGRAL #1

$$\int_0^1 \left(x^6 + 2x^5 + 4x^3 + 5x^2\right) \mathrm{d}x$$

2:00

INTEGRAL #1

$$\int_0^1 \left(x^6 + 2x^5 + 4x^3 + 5x^2\right) \mathrm{d}x$$

$$= \left[\frac{x^7}{7} + \frac{x^6}{3} + x^4 + \frac{5x^3}{3}\right]_0^1$$

$$= \left(\frac{1}{7} + \frac{1}{3} + 1 + \frac{5}{3}\right) - 0$$

$$= \boxed{\frac{22}{7}}$$

READY,

GET SET,...

2:00

INTEGRAL #2

$$\int_0^{\pi/2} (2\cos x - \sin 2x)\,\mathrm{d}x$$

2:00

INTEGRAL #2

$$\int_0^{\pi/2} (2\cos x - \sin 2x)\,\mathrm{d}x$$

$$= \left[2\sin x + \frac{1}{2}\cos 2x\right]_0^{\pi/2}$$

$$= \left(2\sin\frac{\pi}{2} + \frac{1}{2}\cos\pi\right) - \left(2\sin 0 + \frac{1}{2}\cos 0\right)$$

$$= \left(2 - \frac{1}{2}\right) - \left(0 + \frac{1}{2}\right)$$

$$= \boxed{1}$$

READY,

GET SET,...

2:00

INTEGRAL #3

$$\int_4^5 x\sqrt{25-x^2}\,\mathrm{d}x$$

2:00

INTEGRAL #3

$$\int_4^5 x\sqrt{25-x^2}\,\mathrm{d}x$$

$$= -\frac{1}{2}\int_9^0 \sqrt{u}\,\mathrm{d}u \qquad u = 25 - x^2, \quad \mathrm{d}u = -2x\,\mathrm{d}x$$

$$= -\frac{1}{2}\left[\frac{2u^{3/2}}{3}\right]_9^0$$

$$= -\frac{1}{2}\left(0 - \frac{2\cdot 27}{3}\right)$$

$$= \boxed{9}$$

INTEGRAL #4

READY,

GET SET,...

2:00

INTEGRAL #4

$$\int_0^{\pi/3} \left(\sin^2 x + \cos^2 x + \tan^2 x + \sec^2 x\right) dx$$

2:00

INTEGRAL #4

$$\int_0^{\pi/3} \left(\sin^2 x + \cos^2 x + \tan^2 x + \sec^2 x\right) \mathrm{d}x$$

$$= \int_0^{\pi/3} \left(1 + \tan^2 x + \sec^2 x\right) \mathrm{d}x \qquad \sin^2 x + \cos^2 x = 1$$

$$= \int_0^{\pi/3} \left(\sec^2 x + \sec^2 x\right) \mathrm{d}x \qquad 1 + \tan^2 x = \sec^2 x$$

$$= \int_0^{\pi/3} 2\sec^2 x \, \mathrm{d}x$$

$$= \Big[2\tan x\Big]_0^{\pi/3} = \boxed{2\sqrt{3}}$$

READY,

GET SET,...

2:00

INTEGRAL #5

$$\int_0^{(\ln 6)/6} e^x \cdot e^{2x} \cdot e^{3x} \, dx$$

2:00

INTEGRAL #5

$$\int_0^{(\ln 6)/6} e^x \cdot e^{2x} \cdot e^{3x}\, dx$$

$$= \int_0^{(\ln 6)/6} e^{x+2x+3x}\, dx$$

$$= \int_0^{(\ln 6)/6} e^{6x}\, dx$$

$$= \left[\frac{e^{6x}}{6}\right]_0^{(\ln 6)/6}$$

$$= \frac{e^{\ln 6} - 1}{6} = \boxed{\frac{5}{6}}$$

INTEGRAL #6

READY,

GET SET,...

2:00

INTEGRAL #6

$$\int_0^{\pi/7} \frac{\sin 7x}{2-\cos 7x}\,\mathrm{d}x$$

2:00

INTEGRAL #6

$$\int_0^{\pi/7} \frac{\sin 7x}{2-\cos 7x}\,\mathrm{d}x$$

$$= \frac{1}{7}\int_1^3 \frac{1}{u}\,\mathrm{d}u \qquad u = 2-\cos 7x, \quad \mathrm{d}u = 7\sin 7x\,\mathrm{d}x$$

$$= \frac{1}{7}\Big[\ln u\Big]_1^3$$

$$= \frac{1}{7}(\ln 3 - \ln 1)$$

$$= \boxed{\frac{\ln 3}{7} = \ln 3^{1/7} = \ln \sqrt[7]{3}}$$

INTEGRAL #7

READY,

GET SET,...

2:00

INTEGRAL #7

$$\int_0^1 (x+1)(x^4-x^3+x^2-x+1)\,\mathrm{d}x$$

2:00

INTEGRAL #7

$$\int_0^1 (x+1)(x^4 - x^3 + x^2 - x + 1)\,\mathrm{d}x$$

$$= \int_0^1 \left(x^5 + 1\right)\,\mathrm{d}x$$

$$= \left[\frac{x^6}{6} + x\right]_0^1$$

$$= \left(\frac{1}{6} + 1\right) - 0$$

$$= \boxed{\frac{7}{6}}$$

INTEGRAL #8

READY,

GET SET,...

2:00

INTEGRAL #8

$$\int_{\ln(\pi/6)}^{\ln(\pi/2)} e^x \sin^2(e^x) \cos(e^x)\, dx$$

2:00

INTEGRAL #8

$$\int_{\ln(\pi/6)}^{\ln(\pi/2)} e^x \sin^2(e^x)\cos(e^x)\,dx$$

$$= \int_{1/2}^{1} u^2\,du \qquad u = \sin(e^x),\quad du = e^x\cos(e^x)\,dx$$

$$= \left[\frac{u^3}{3}\right]_{1/2}^{1}$$

$$= \frac{1}{3} - \frac{1}{24}$$

$$= \boxed{\frac{7}{24}}$$

READY,

GET SET,...

2:00

INTEGRAL #9

$$\int_1^2 \frac{x^6 + x^3 - 1}{x^3} \, \mathrm{d}x$$

2:00

INTEGRAL #9

$$\int_1^2 \frac{x^6 + x^3 - 1}{x^3}\,\mathrm{d}x$$

$$= \int_1^2 \left(\frac{x^6}{x^3} + \frac{x^3}{x^3} - \frac{1}{x^3}\right)\mathrm{d}x$$

$$= \int_1^2 \left(x^3 + 1 - \frac{1}{x^3}\right)\mathrm{d}x$$

$$= \left[\frac{x^4}{4} + x + \frac{1}{2x^2}\right]_1^2$$

$$= \boxed{\frac{35}{8}}$$

READY,

GET SET,...

2:00

INTEGRAL #10

$$\int_0^{\pi/4} \left(\tan^{2018} x + \tan^{2016} x\right) \mathrm{d}x$$

2:00

INTEGRAL #10

$$\int_0^{\pi/4} \left(\tan^{2018} x + \tan^{2016} x\right) \mathrm{d}x$$

$$= \int_0^{\pi/4} \tan^{2016} x \left(\tan^2 x + 1\right) \mathrm{d}x$$

$$= \int_0^{\pi/4} \tan^{2016} x \sec^2 x \, \mathrm{d}x \qquad \tan^2 x + 1 = \sec^2 x$$

$$= \int_0^{1} u^{2016} \, \mathrm{d}u \qquad u = \tan x, \quad \mathrm{d}u = \sec^2 x \, \mathrm{d}x$$

$$= \left[\frac{u^{2017}}{2017}\right]_0^1 = \boxed{\frac{1}{2017}}$$

READY,

GET SET,...

2:00

INTEGRAL #11

$$\int_0^1 \sqrt[4]{x \cdot \sqrt[3]{x \cdot \sqrt{x}}}\,\mathrm{d}x$$

2:00

INTEGRAL #11

$$\int_0^1 \sqrt[4]{x \cdot \sqrt[3]{x \cdot \sqrt{x}}}\,\mathrm{d}x$$

$$= \int_0^1 \left(x \cdot \left(x \cdot x^{1/2}\right)^{1/3}\right)^{1/4} \mathrm{d}x$$

$$= \int_0^1 \left(x \cdot \left(x^{3/2}\right)^{1/3}\right)^{1/4} \mathrm{d}x$$

$$= \int_0^1 \left(x \cdot x^{1/2}\right)^{1/4} \mathrm{d}x$$

$$= \int_0^1 \left(x^{3/2}\right)^{1/4} \mathrm{d}x = \int_0^1 x^{3/8}\,\mathrm{d}x = \left[\frac{8x^{11/8}}{11}\right]_0^1 = \boxed{\frac{8}{11}}$$

READY,

GET SET,...

2:00

INTEGRAL #12

$$\int_0^{\pi/4} x \cos 2x \, \mathrm{d}x$$

2:00

INTEGRAL #12

$$\int_0^{\pi/4} x\cos 2x \,\mathrm{d}x$$

Integrate by parts: $u = x \quad \mathrm{d}v = \cos 2x\,\mathrm{d}x$

$\mathrm{d}u = \mathrm{d}x \quad v = \frac{1}{2}\sin 2x$

$$= \left[\frac{x\sin 2x}{2}\right]_0^{\pi/4} - \frac{1}{2}\int_0^{\pi/4} \sin 2x\,\mathrm{d}x$$

$$= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_0^{\pi/4}$$

$$= \boxed{\frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}}$$

READY,

GET SET,...

2:00

INTEGRAL #13

$$\int_1^2 (x-5)^2(x-1)^7\,\mathrm{d}x$$

2:00

INTEGRAL #13

$$\int_1^2 (x-5)^2(x-1)^7\,\mathrm{d}x$$

$$= \int_0^1 (u-4)^2 u^7\,\mathrm{d}u \qquad u = x-1, \;\; u-4 = x-5, \;\; \mathrm{d}u = \mathrm{d}x$$

$$= \int_0^1 \left(u^9 - 8u^8 + 16u^7\right)\mathrm{d}u$$

$$= \left[\frac{u^{10}}{10} - \frac{8u^9}{9} + 2u^8\right]_0^1$$

$$= \boxed{\frac{109}{90}}$$

READY,

GET SET,...

2:00

INTEGRAL #14

$$\int_{\pi^2/16}^{\pi^2/4} \frac{\sin\sqrt{x} + \cos\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x$$

2:00

INTEGRAL #14

$$\int_{\pi^2/16}^{\pi^2/4} \frac{\sin\sqrt{x}+\cos\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x$$

$$= 2\int_{\pi/4}^{\pi/2} (\sin u + \cos u)\,\mathrm{d}u \qquad u=\sqrt{x},\quad \mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$$

$$= 2\Big[-\cos u + \sin u\Big]_{\pi/4}^{\pi/2}$$

$$= 2\left[(0+1)-\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)\right]$$

$$= \boxed{2}$$

READY,

GET SET,...

2:00

INTEGRAL #15

$$\int_0^1 (x-2)(x-1)(x+1)(x+2)\,\mathrm{d}x$$

2:00

INTEGRAL #15

$$\int_0^1 (x-2)(x-1)(x+1)(x+2)\,\mathrm{d}x$$

$$= \int_0^1 (x^2-4)(x^2-1)\,\mathrm{d}x$$

$$= \int_0^1 (x^4-5x^2+4)\,\mathrm{d}x$$

$$= \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1$$

$$= \boxed{\frac{38}{15}}$$