HIGH SCHOOL FINAL ROUND

The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

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HIGH SCHOOL FINAL ROUND

Contestants must circle their final answer.
Contestants do **not** need to include the constant of integration $+C$ in their answer.

The last person remaining wins an additional $75 and will be crowned the **Integration Champion**!

INTEGRAL #1

READY,

GET SET,...

2:30

INTEGRAL #1

$$\int \sqrt[3]{5x+7}\,\mathrm{d}x$$

2:30

INTEGRAL #1

$$\int \sqrt[3]{5x+7}\,\mathrm{d}x$$

$$= \frac{1}{5}\int u^{1/3}\,\mathrm{d}u \qquad u = 5x+7, \quad \mathrm{d}u = 5\,\mathrm{d}x$$

$$= \frac{1}{5}\cdot\frac{3u^{4/3}}{4} + C$$

$$= \boxed{\frac{3(5x+7)^{4/3}}{20} + C}$$

INTEGRAL #2

READY,

GET SET,...

2:30

INTEGRAL #2

$$\int \left(8\sin^3 x - 3\sin^2 x + 6\sin x\right)\cos x \, dx$$

2:30

INTEGRAL #2

$$\int \left(8\sin^3 x - 3\sin^2 x + 6\sin x\right)\cos x \, dx$$

$$= \int \left(8u^3 - 3u^2 + 6u\right) du \qquad u = \sin x, \quad du = \cos x \, dx$$

$$= 2u^4 - u^3 + 3u^2 + C$$

$$= \boxed{2\sin^4 x - \sin^3 x + 3\sin^2 x + C}$$

INTEGRAL #3

READY,

GET SET,...

2:30

INTEGRAL #3

$$\int \frac{\left(\sqrt{x}+\sqrt{2}\right)^5}{\sqrt{x}}\,dx$$

2:30

INTEGRAL #3

$$\int \frac{\left(\sqrt{x}+\sqrt{2}\right)^5}{\sqrt{x}}\,dx$$

$$= 2\int u^5\,du \qquad u = \sqrt{x}+\sqrt{2}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= \frac{u^6}{3} + C$$

$$= \boxed{\frac{1}{3}\left(\sqrt{x}+\sqrt{2}\right)^6 + C}$$

INTEGRAL #4

READY,

GET SET,...

2:30

INTEGRAL #4

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx$$

2:30

INTEGRAL #4

$$\int e^{2x}\sqrt{1+e^{2x}}\,dx$$

$$= \frac{1}{2}\int \sqrt{u}\,du \qquad u = 1+e^{2x}, \quad du = 2e^{2x}\,dx$$

$$= \frac{1}{2}\cdot\frac{2u^{3/2}}{3} + C$$

$$= \boxed{\frac{(1+e^{2x})^{3/2}}{3} + C}$$

INTEGRAL #5

READY,

GET SET,...

2:30

INTEGRAL #5

$$\int (x-1)\left(x^2+x+1+\frac{1}{x}\right)\,\mathrm{d}x$$

2:30

INTEGRAL #5

$$\int (x-1)\left(x^2+x+1+\frac{1}{x}\right)\,dx$$

$$= \int \left(x^3-\frac{1}{x}\right)\,dx$$

$$= \boxed{\frac{x^4}{4}-\ln|x|+C}$$

INTEGRAL #6

READY,

GET SET,...

2:30

INTEGRAL #6

$$\int (x + \cos 2x)\left(x^2 + \sin 2x\right)^3 \, \mathrm{d}x$$

2:30

INTEGRAL #6

$$\int (x+\cos 2x)\left(x^2+\sin 2x\right)^3\,dx$$

$$= \frac{1}{2}\int u^3\,du \qquad u = x^2+\sin 2x, \quad du = 2(x+\cos 2x)\,dx$$

$$= \frac{1}{2}\cdot\frac{u^4}{4}+C$$

$$= \boxed{\frac{(x^2+\sin 2x)^4}{8}+C}$$

INTEGRAL #7

READY,

GET SET,...

2:30

INTEGRAL #7

$$\int (4x-3)e^{x/2}\,dx$$

2:30

INTEGRAL #7

$$\int (4x-3)e^{x/2}\,dx$$

Integrate by parts: $u = 4x-3 \quad dv = e^{x/2}\,dx$

$du = 4\,dx \qquad v = 2e^{x/2}$

$$= 2(4x-3)e^{x/2} - 8\int e^{x/2}\,dx$$

$$= \boxed{2(4x-3)e^{x/2} - 16e^{x/2} + C \text{ or } 2(4x-11)e^{x/2} + C}$$

INTEGRAL #8

READY,

GET SET,...

2:30

INTEGRAL #8

$$\int (x^2+2)\sin\left(x^3+6x+7\right)\cos\left(x^3+6x+7\right)\,dx$$

2:30

INTEGRAL #8

$$\int (x^2+2)\sin\left(x^3+6x+7\right)\cos\left(x^3+6x+7\right)\,dx$$

$$u=\sin\left(x^3+6x+7\right),\quad du=3(x^2+2)\cos\left(x^3+6x+7\right)\,dx$$

$$=\frac{1}{3}\int u\,du$$

$$=\frac{u^2}{6}+C$$

$$=\boxed{\frac{\sin^2\left(x^3+6x+7\right)}{6}+C \quad \text{or} \quad -\frac{\cos^2\left(x^3+6x+7\right)}{6}+C}$$

INTEGRAL #9

READY,

GET SET,...

2:30

INTEGRAL #9

$$\int \tan x \sec^4 x \, dx$$

2:30

INTEGRAL #9

$$\int \tan x \sec^4 x \, dx$$

$$= \int \sec^3 x \cdot \sec x \tan x \, dx$$

$$= \int u^3 \, du \qquad u = \sec x, \quad du = \sec x \tan x \, dx$$

$$= \frac{u^4}{4} + C$$

$$= \boxed{\frac{\sec^4 x}{4} + C}$$

INTEGRAL #10

READY,

GET SET,...

2:30

INTEGRAL #10

$$\int \left(\frac{x}{3}+\frac{x^3}{3}+\frac{x^5}{3}\right)\left(\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}\right) \mathrm{d}x$$

2:30

INTEGRAL #10

$$\int \left(\frac{x}{3}+\frac{x^3}{3}+\frac{x^5}{3}\right)\left(\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}\right)\mathrm{d}x$$

$$= \frac{1}{3}\int u\,\mathrm{d}u \qquad u = \frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}, \quad \mathrm{d}u = \left(x+x^3+x^5\right)\mathrm{d}x$$

$$= \frac{1}{6}\cdot u^2 + C$$

$$= \boxed{\frac{1}{6}\left(\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}\right)^2 + C \quad \text{or} \quad \frac{x^{12}}{216}+\frac{x^{10}}{72}+\frac{11x^8}{288}+\frac{x^6}{24}+\frac{x^4}{24}+C}$$

INTEGRAL #11

READY,

GET SET,...

2:30

INTEGRAL #11

$$\int \csc\frac{x}{2} \cdot \sec\frac{x}{2} \cdot \sin\frac{x}{2} \cdot \tan\frac{x}{2}\,dx$$

2:30

INTEGRAL #11

$$\int \csc\frac{x}{2}\cdot\sec\frac{x}{2}\cdot\sin\frac{x}{2}\cdot\tan\frac{x}{2}\,dx$$

$$= \int \sec\frac{x}{2}\cdot\tan\frac{x}{2}\,dx \qquad \csc\theta\cdot\sin\theta = 1$$

$$= 2\int \sec u\tan u\,du \qquad u = \frac{x}{2},\quad du = \frac{1}{2}dx$$

$$= 2\sec u + C$$

$$= \boxed{2\sec\frac{x}{2} + C}$$

INTEGRAL #12

READY,

GET SET,...

2:30

INTEGRAL #12

$$\int \frac{x+2019}{(x-2000)^3}\,\mathrm{d}x$$

2:30

INTEGRAL #12

$$\int \frac{x+2019}{(x-2000)^3}\,dx$$

$$= \int \frac{u+4019}{u^3}\,du \qquad u = x-2000, \quad x = u+2000, \quad dx = du$$

$$= \int \left(\frac{1}{u^2} + \frac{4019}{u^3}\right) dx$$

$$= -\frac{1}{u} - \frac{4019}{2u^2} + C$$

$$= \boxed{-\frac{1}{x-2000} - \frac{4019}{2(x-2000)^2} + C \quad \text{or} \quad -\frac{2x+19}{2(x-2000)^2} + C}$$

INTEGRAL #13

READY,

GET SET,...

2:30

INTEGRAL #13

$$\int \frac{e^{2x}}{4e^{4x} + 4e^{2x} + 1} \, dx$$

2:30

INTEGRAL #13

$$\int \frac{e^{2x}}{4e^{4x}+4e^{2x}+1}\,dx$$

$$= \int \frac{e^{2x}}{(2e^{2x}+1)^2}\,dx$$

$$= \frac{1}{4}\int \frac{1}{u^2}\,du \qquad u = 2e^{2x}+1, \quad du = 4e^{2x}\,dx$$

$$= -\frac{1}{4u} + C$$

$$= \boxed{-\frac{1}{4(2e^{2x}+1)} + C}$$

INTEGRAL #14

READY,

GET SET,...

2:30

INTEGRAL #14

$$\int (1 + \csc x \cot x)(1 - \csc x \cot x)\, dx$$

2:30

INTEGRAL #14

$$\int (1+\csc x \cot x)(1-\csc x \cot x)\,dx$$

$$= \int \left(1-\csc^2 x \cot^2 x\right) dx$$

$$= \int 1\,dx - \int \csc^2 x \cot^2 x\,dx$$

$$= x + \int u^2\,du \qquad u = \cot x, \quad du = -\csc^2 x\,dx$$

$$= \boxed{x + \frac{\cot^3 x}{3} + C}$$

INTEGRAL #15

READY,

GET SET,...

2:30

INTEGRAL #15

$$\int \frac{x^2}{8x^3 + 13x^{-3}} \, dx$$

2:30

INTEGRAL #15

$$\int \frac{x^2}{8x^3 + 13x^{-3}}\,dx$$

$$= \int \frac{x^2}{8x^3 + 13x^{-3}} \cdot \frac{x^3}{x^3}\,dx$$

$$= \int \frac{x^5}{8x^6 + 13}\,dx$$

$$= \frac{1}{48}\int \frac{1}{u}\,du \qquad u = 8x^6 + 13, \quad du = 48x^5\,dx$$

$$= \boxed{\frac{1}{48}\ln\left(8x^6 + 13\right) + C}$$

INTEGRAL #16

READY,

GET SET,...

2:30

INTEGRAL #16

$$\int \frac{\sin 4x}{\sin 2x}\,\mathrm{d}x$$

2:30

INTEGRAL #16

$$\int \frac{\sin 4x}{\sin 2x}\,dx$$

$$= \int \frac{2\sin 2x \cos 2x}{\sin 2x}\,dx \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$= \int 2\cos 2x\,dx$$

$$= \boxed{\sin 2x + C}$$

INTEGRAL #17

READY,

GET SET,...

2:30

INTEGRAL #17

$$\int (\sec x + \tan x)^2 \, dx$$

2:30

INTEGRAL #17

$$\int (\sec x + \tan x)^2 \, dx$$

$$= \int \left(\sec^2 x + 2 \sec x \tan x + \tan^2 x\right) dx$$

$$= \int \left(\sec^2 x + 2 \sec x \tan x + \sec^2 x - 1\right) dx$$

$$= \int \left(2 \sec^2 x + 2 \sec x \tan x - 1\right) dx$$

$$= \boxed{2 \tan x + 2 \sec x - x + C}$$

INTEGRAL #18

READY,

GET SET,...

2:30

INTEGRAL #18

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \, dx$$

2:30

INTEGRAL #18

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u}\,du \qquad u = e^{2x} + e^{-2x}, \quad du = 2(e^{2x} - e^{-2x})\,dx$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{1}{2}\ln\left(e^{2x} + e^{-2x}\right) + C}$$

INTEGRAL #19

READY,

GET SET,...

2:30

INTEGRAL #19

$$\int \sqrt{x + x^2\sqrt{x}}\,\mathrm{d}x$$

2:30

INTEGRAL #19

$$\int \sqrt{x + x^2\sqrt{x}}\,dx$$

$$= \int \sqrt{x} \cdot \sqrt{1 + x\sqrt{x}}\,dx$$

$$= \frac{2}{3}\int \sqrt{u}\,du \qquad u = 1 + x\sqrt{x}, \quad du = \frac{3}{2}\sqrt{x}\,dx$$

$$= \frac{2}{3} \cdot \frac{2}{3} \cdot u^{3/2} + C$$

$$= \boxed{\frac{4}{9}\left(1 + x\sqrt{x}\right)^{3/2} + C}$$

INTEGRAL #20

READY,

GET SET,...

2:30

INTEGRAL #20

$$\int \frac{x-1}{x^2 - x \ln x} \, dx$$

2:30

INTEGRAL #20

$$\int \frac{x-1}{x^2 - x\ln x}\,dx$$

$$= \int \frac{x-1}{x(x-\ln x)}\,dx$$

$$= \int \frac{1}{u}\,du \qquad u = x - \ln x, \quad du = \left(1 - \frac{1}{x}\right)dx = \frac{x-1}{x}\,dx$$

$$= \ln|u| + C$$

$$= \boxed{\ln(x - \ln x) + C}$$

INTEGRAL #21

READY,

GET SET,...

2:30

INTEGRAL #21

$$\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$$

2:30

INTEGRAL #21

$$\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$$

$$= \int \frac{2u-2}{u}\,\mathrm{d}u \qquad u = 1+\sqrt{x},\ \ (u-1)^2 = x,\ \ (2u-2)\,\mathrm{d}u = \mathrm{d}x$$

$$= \int \left(2 - \frac{2}{u}\right)\mathrm{d}u$$

$$= 2u - 2\ln|u| + C$$

$$= \boxed{2\left(1+\sqrt{x}\right) - 2\ln\left(1+\sqrt{x}\right) + C \ \text{ or } \ 2\sqrt{x} - 2\ln\left(1+\sqrt{x}\right) + C}$$

INTEGRAL #22

READY,

GET SET,...

2:30

INTEGRAL #22

$$\int \frac{\sin 2x}{\sqrt{3+4\sin^2 x}}\,dx$$

2:30

INTEGRAL #22

$$\int \frac{\sin 2x}{\sqrt{3+4\sin^2 x}}\,dx$$

$$= \int \frac{2\sin x\cos x}{\sqrt{3+4\sin^2 x}}\,dx \qquad \text{Trig identity: } \sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{u}}\,du \qquad u = 3+4\sin^2 x, \quad du = 8\sin x\cos x\,dx$$

$$= \frac{1}{2}\sqrt{u} + C$$

$$= \boxed{\frac{1}{2}\sqrt{3+4\sin^2 x} + C}$$

INTEGRAL #23

READY,

GET SET,...

2:30

INTEGRAL #23

$$\int e^{\sqrt{x}}\,dx$$

2:30

INTEGRAL #23

$$\int e^{\sqrt{x}}\,dx$$

$$= \int 2ue^{u}\,du \qquad u = \sqrt{x},\ \ u^2 = x,\ \ 2u\,du = dx$$

$$= 2ue^{u} - \int 2e^{u}\,du \qquad \text{Integration by parts}$$

$$= 2ue^{u} - 2e^{u} + C$$

$$= \boxed{2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$$

INTEGRAL #24

READY,

GET SET,...

2:30

INTEGRAL #24

$$\int x^3 e^{x^2} \, dx$$

2:30

INTEGRAL #24

$$\int x^3 e^{x^2}\,dx$$

$$= \int x^2 \cdot x e^{x^2}\,dx$$

$$= \frac{1}{2}\int u e^u\,du \qquad u = x^2, \quad du = 2x\,dx$$

$$= \frac{1}{2}(u e^u - e^u) + C \qquad \text{Integration by parts}$$

$$= \boxed{\frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C}$$

INTEGRAL #25

READY,

GET SET,...

2:30

INTEGRAL #25

$$\int \frac{\ln \ln x \cdot \ln x}{x} \, dx$$

2:30

INTEGRAL #25

$$\int \frac{\ln\ln x \cdot \ln x}{x}\,dx$$

$$= \int u \ln u \,du \qquad u = \ln x, \quad du = \frac{1}{x}\,dx$$

$$= \frac{u^2 \ln u}{2} - \int \frac{u}{2}\,du \qquad \text{Integration by parts}$$

$$= \frac{u^2 \ln u}{2} - \frac{u^2}{4} + C$$

$$= \boxed{\frac{\ln^2 x \cdot \ln\ln x}{2} - \frac{\ln^2 x}{4} + C}$$