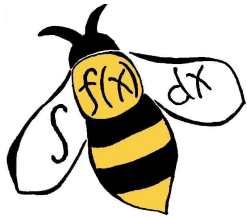


HIGH SCHOOL ROUND ONE



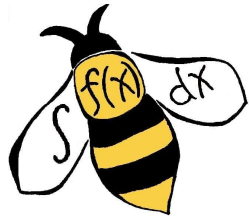
You will have **two minutes** to evaluate each of the fifteen definite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, you must do so before the next problem is displayed by notifying a grader.

For each correct answer, you will receive one raffle ticket to be entered for prizes that will be drawn after dinner.



HIGH SCHOOL ROUND ONE



All answers must be simplified which means

- Rational numbers must be simplified and reduced.

$$\frac{4}{2} \text{ X vs } 2 \text{ ✓}$$

$$\frac{1}{3} + \frac{1}{6} \text{ X vs } \frac{1}{2} \text{ ✓}$$

- Trig functions must be evaluated when possible.

$$\sin \frac{\pi}{3} \text{ X vs } \frac{\sqrt{3}}{2} \text{ ✓}$$

$$\tan 0 \text{ X vs } 0 \text{ ✓}$$

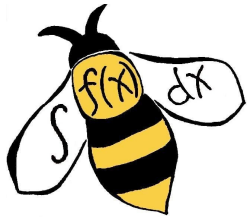
- You do **not** need to combine multiple logs or fractions with irrational numbers (e.g. π , e , $\ln 3$).

$$\frac{1}{e} + 2 \text{ ✓, } \frac{1 + 2e}{e} \text{ ✓}$$

$$2 \ln 3 \text{ ✓, } \ln 9 \text{ ✓}$$



HIGH SCHOOL ROUND ONE



- However, **no** negative powers should remain.

$$e^{-1} \text{ X vs } \frac{1}{e} \text{ ✓}$$

At most five participants will move onto the finals – to be determined by the total number of correct answers and tiebreaking criteria if necessary.

Everyone moving onto the finals will receive \$25.

Any questions?

**READY,
GET SET,...**

2:00

INTEGRAL #1

$$\int_{-1}^1 (5x^3 + 6x^2 + 7x + 8) dx$$

INTEGRAL #1

$$\int_{-1}^1 (5x^3 + 6x^2 + 7x + 8) dx$$

$$= \left[\frac{5x^4}{4} + 2x^3 + \frac{7x^2}{2} + 8x \right]_{-1}^1$$

$$= \left(\frac{5}{4} + 2 + \frac{7}{2} + 8 \right) - \left(\frac{5}{4} - 2 + \frac{7}{2} - 8 \right)$$

$$= \boxed{20}$$

**READY,
GET SET,...**

2:00

INTEGRAL #2

$$\int_0^{\pi/3} (3 \sin x - \cos 3x) dx$$

INTEGRAL #2

$$\begin{aligned} & \int_0^{\pi/3} (3 \sin x - \cos 3x) dx \\ &= \left[-3 \cos x - \frac{1}{3} \sin 3x \right]_0^{\pi/3} \\ &= \left(-3 \cdot \frac{1}{2} - \frac{1}{3} \cdot 0 \right) - \left(-3 \cdot 1 - \frac{1}{3} \cdot 0 \right) \\ &= \boxed{\frac{3}{2}} \end{aligned}$$

**READY,
GET SET,...**

2:00

INTEGRAL #3

$$\int_1^2 \frac{3x + 4}{x^5} dx$$

INTEGRAL #3

$$\int_1^2 \frac{3x + 4}{x^5} dx$$

$$= \int_1^2 \left(\frac{3x}{x^5} + \frac{4}{x^5} \right) dx = \int_1^2 \left(\frac{3}{x^4} + \frac{4}{x^5} \right) dx$$

$$= \left[-\frac{1}{x^3} - \frac{1}{x^4} \right]_1^2$$

$$= \left(-\frac{1}{8} - \frac{1}{16} \right) - (-1 - 1)$$

$$= \boxed{\frac{29}{16}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #4

$$\int_0^{\pi/6} \frac{18 \cos x}{(1 + \sin x)^3} dx$$

INTEGRAL #4

$$\int_0^{\pi/6} \frac{18 \cos x}{(1 + \sin x)^3} dx$$

$$= 18 \int_1^{3/2} \frac{1}{u^3} du \quad u = 1 + \sin x, \quad du = \cos x dx$$

$$= -9 \cdot \left[\frac{1}{u^2} \right]_1^{3/2}$$

$$= -9 \cdot \left(\frac{4}{9} - 1 \right)$$

$$= \boxed{5}$$

**READY,
GET SET,...**

2:00

INTEGRAL #5

$$\int_1^2 (3x + \sqrt{5})(3x - \sqrt{5}) dx$$

INTEGRAL #5

$$\int_1^2 (3x + \sqrt{5})(3x - \sqrt{5}) dx$$

$$= \int_1^2 (9x^2 - 5) dx$$

$$= [3x^3 - 5x]_1^2$$

$$= 16$$

**READY,
GET SET,...**

2:00

INTEGRAL #6

$$\int_1^e 4x \ln x \, dx$$

INTEGRAL #6

$$\int_1^e 4x \ln x \, dx$$

Integration by parts:

$$u = \ln x \quad dv = 4x \, dx$$
$$du = \frac{1}{x} \, dx \quad v = 2x^2$$

$$= \left[2x^2 \ln x \right]_1^e - \int_1^e 2x \, dx$$

$$= \left[2x^2 \ln x - x^2 \right]_1^e$$

$$= \boxed{e^2 + 1}$$

**READY,
GET SET,...**

2:00

INTEGRAL #7

$$\int_{-2}^0 \frac{x-1}{x^2-2x-9} dx$$

INTEGRAL #7

$$\int_{-2}^0 \frac{x-1}{x^2-2x-9} dx$$

$$= \frac{1}{2} \int_{-1}^{-9} \frac{1}{u} du \quad u = x^2 - 2x - 9, \quad du = 2(x-1) dx$$

$$= \left[\frac{1}{2} \ln|u| \right]_{-1}^{-9}$$

$$= \frac{1}{2} \ln 9 \text{ or } \ln 3$$

**READY,
GET SET,...**

2:00

INTEGRAL #8

$$\int_0^{\pi/3} \frac{\sin x}{4 + \sin^2 x + \cos^2 x} dx$$

INTEGRAL #8

$$\begin{aligned} & \int_0^{\pi/3} \frac{\sin x}{4 + \sin^2 x + \cos^2 x} dx \\ &= \int_0^{\pi/3} \frac{\sin x}{5} dx \quad \sin^2 \theta + \cos^2 \theta = 1 \\ &= \left[-\frac{\cos x}{5} \right]_0^{\pi/3} \\ &= -\frac{1}{10} + \frac{1}{5} \\ &= \boxed{\frac{1}{10}} \end{aligned}$$

**READY,
GET SET,...**

2:00

INTEGRAL #9

$$\int_0^1 \left(\sqrt[4]{x^9} + \sqrt[5]{x^8} + \sqrt[6]{x^7} \right) dx$$

INTEGRAL #9

$$\begin{aligned} & \int_0^1 \left(\sqrt[4]{x^9} + \sqrt[5]{x^8} + \sqrt[6]{x^7} \right) dx \\ &= \int_0^1 \left(x^{9/4} + x^{8/5} + x^{7/6} \right) dx \\ &= \left[\frac{4x^{13/4}}{13} + \frac{5x^{13/5}}{13} + \frac{6x^{13/6}}{13} \right]_0^1 \\ &= \boxed{\frac{15}{13}} \end{aligned}$$

**READY,
GET SET,...**

2:00

INTEGRAL #10

$$\int_{\pi/6}^{\pi/2} \frac{\sin x \cos x}{(1 + \cos^2 x)^2} dx$$

INTEGRAL #10

$$\int_{\pi/6}^{\pi/2} \frac{\sin x \cos x}{(1 + \cos^2 x)^2} dx$$

$$= -\frac{1}{2} \int_{7/4}^1 \frac{1}{u^2} du \quad u = 1 + \cos^2 x, \quad du = -2 \cos x \sin x dx$$

$$= \left[\frac{1}{2u} \right]_{7/4}^1$$

$$= \boxed{\frac{3}{14}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #11

$$\int_{1/2}^{4/3} \frac{1}{x^2} \sqrt{\frac{x+4}{x}} dx$$

INTEGRAL #11

$$\int_{1/2}^{4/3} \frac{1}{x^2} \sqrt{\frac{x+4}{x}} dx$$

$$= -\frac{1}{4} \int_9^4 \sqrt{u} du$$

$$u = \frac{x+4}{x} = 1 + \frac{4}{x}, \quad du = -\frac{4}{x^2} dx$$

$$= -\frac{1}{4} \left[\frac{2u^{3/2}}{3} \right]_9^4$$

$$= \boxed{\frac{19}{6}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #12

$$\int_1^4 \left(x + \frac{1}{x} \right)^2 dx$$

INTEGRAL #12

$$\int_1^4 \left(x + \frac{1}{x}\right)^2 dx$$

$$= \int_1^4 \left(x^2 + 2 + \frac{1}{x^2}\right) dx$$

$$= \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]_1^4$$

$$= \boxed{\frac{111}{4}}$$

**READY,
GET SET,...**

2:00

INTEGRAL #13

$$\int_1^2 e^{3 \ln x} dx$$

INTEGRAL #13

$$\begin{aligned} & \int_1^2 e^{3\ln x} dx \\ &= \int_1^2 e^{\ln x^3} dx = \int_1^2 x^3 dx \\ &= \left[\frac{x^4}{4} \right]_1^2 \\ &= \boxed{\frac{15}{4}} \end{aligned}$$

**READY,
GET SET,...**

2:00

INTEGRAL #14

$$\int_0^2 \left(5 + x(4 + x(3 + x)) \right) dx$$

INTEGRAL #14

$$\begin{aligned} & \int_0^2 (5 + x(4 + x(3 + x))) dx \\ &= \int_0^2 (x^3 + 3x^2 + 4x + 5) dx \\ &= \left[\frac{x^4}{4} + x^3 + 2x^2 + 5x \right]_0^2 \\ &= \boxed{30} \end{aligned}$$

**READY,
GET SET,...**

2:00

INTEGRAL #15

$$\int_0^{\sqrt{\pi/3}} \frac{x}{\cos^2(x^2)} dx$$

INTEGRAL #15

$$\int_0^{\sqrt{\pi/3}} \frac{x}{\cos^2(x^2)} dx$$

$$= \int_0^{\sqrt{\pi/3}} x \sec^2(x^2) dx$$

$$= \frac{1}{2} \int_0^{\pi/3} \sec^2 u du \quad u = x^2, \quad du = 2x dx$$

$$= \left[\frac{1}{2} \tan u \right]_0^{\pi/3} = \boxed{\frac{\sqrt{3}}{2}}$$